EFFECTS OF MHD CHANNEL PROFILING ON DIAGONAL-GENERATOR INTEGRAL CHARACTERISTICS

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A transport MHD system is used to show that slight narrowing in the electrode zone (about 10°) gives a substantial improvement in the integral characteristics.

The section of an MHD channel may be profiled to equalize the Hall emf's and to reduce the parasitic currents when there is plasma inhomogeneity, and the effects on the local characteristics of an MHD generator have been considered in [1].

A one-dimensional electrodynamic model has been used for a nonrectangular channel, where the inhomogeneity parameter is

$$G_{\mathrm{B}} = \langle \sigma z_* \rangle \left\langle \frac{1+\beta^2}{\sigma z_*} \right\rangle - \langle \beta \rangle^2. \tag{1}$$

Then (1) goes over to the usual relation [2]

$$G_R = \langle \sigma \rangle \left\langle \frac{1+\beta^2}{\sigma} \right\rangle - \langle \beta \rangle^2 \tag{2}$$

when z_* is constant, i.e., for a rectangular cross section. A previous study [1] has shown that one can substantially reduce the parasitic Hall currents and correspondingly the parameter of (1) in a supersonic channel simply by narrowing the section in the electrode zone by an amount equal to the thickness of the boundary layer. We consider that this narrowing is better than other methods of improving the electrodynamic characteristics in that it can be implemented in a simple fashion. Also, a section narrowing towards the electrode walls can approximate to an elliptical one, and such a section has an advantage over a rectangular one in having better gasdynamic characteristics. One can improve the electrodynamic and gasdynamic characteristics of a supersonic MHD channel by examining not the local characteristics, as in [1], but the integral ones. Supersonic flows are usually employed in transport MHD generators. To provide the required specific characteristics, one uses high-enthalpy metallized solid fuel [3], whose properties are described by approximation formulas and Table 1:

TABLE 1. Approximation Coefficients for the Properties of the Working Body

Function	Coefficients						Coefficients		
	A_{00}	A_{10}	A_{20}	A_{01}	A_{02}	A_{12}	A_{21}	A_{11}	A_{22}
$P(H, S)$	−2,29021	10,3359	0,56825	−38,8255	−18,2295	−33,9104	−4,85435	2,39588	−38,2792
$y_1(H)$	−2,29021	—	—	−38,8255	−18,2295	—	—	—	—
$y_2(H)$	10,3359	—	—	2,39588	33,9104	—	—	—	—
$y_3(H)$	0,56825	—	—	−4,85435	−38,2792	—	—	—	—
$z_1(S)$	−2,29021	10,3359	0,56825	—	—	—	—	—	—
$z_2(S)$	−38,8255	2,39588	−4,85435	—	—	—	—	—	—
$z_3(S)$	−18,2295	−33,9104	−38,2792	—	—	—	—	—	—
$T(H, S)$	1,26021	0,554795	—	−1,68397	—	—	—	−1,67802	—
$\rho(H, S, P, T)$	1,1011	−0,282415	—	−0,888447	—	—	—	−1,01969	—
$\sigma(H, S)$	0,585449	1,54089	0,701729	−1,22648	−39,9367	32,2969	42,4601	27,087	59,18
$\beta(H, S)$	−1,08025	−9,22747	—	33,1301	—	—	—	−4,19829	—

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TABLE 2. Calculated Parameters for Rectangular Section

Parameter	Section number							
	1	2	3	4	5	6	7	8
L, m	0	0,68	1,36	2,05	2,73	3,41	4,09	4,78
P, MPa	0,142	0,034	0,021	0,0171	0,0148	0,0132	0,0120	0,0117
$\langle T \rangle$, °K	2766	2463	2383	2343	2312	2291	2283	2272
$\langle U \rangle$, m/sec	2448	2474	2304	2098	1920	1821	1757	1624
$\langle \sigma \rangle$, S/m	43,4	25,4	21,9	20,0	18,4	17,4	17,4	16,8
$\langle \beta \rangle$	0,63	2,53	4,03	5,09	5,88	6,62	7,24	7,44
E_x, V/m	—1916	—2645	—3001	—2750	—2256	—2308	—2799	—2787
γ, deg	39,0	44,1	49,52	55,5	60,13	65,39	70,65	76,0
Q, mW/m^2	0	0,773	1,20	1,52	1,78	1,99	2,18	2,34
F, m^2	0,0043	0,0163	0,0282	0,0401	0,0521	0,064	0,0759	0,088
q, mW	5,79	1,34	0,674	0,405	0,268	0,198	0,160	0,122
y_0, m^2	0,0486	0,182	0,315	0,449	0,582	0,715	0,849	0,982
N, mW	0	0,725	1,70	2,69	3,46	3,80	3,93	4,36
N_V, mW/m^3	0	110	86	68,1	53,1	39,3	29,3	24,5

$$P(H,\ S) = 10^6 \exp\Big[\sum_{i=0,j=0}^{i=2,j=2} A_{ij}\varphi^i\psi^j\Big];$$

while the enthalpy and entropy may be calculated from the auxiliary functions F_1 and F_2, respectively:

$$F_1 = y_2^2 - 4y_1y_3,$$

$$y_1(S,\ P) = \sum_{j=0}^{j=2} A_{0j}\psi^j - \lg(P/10^6),$$

$$y_2(S) = \sum_{j=0}^{j=2} A_{1j}\psi^j \text{ и } y_3(S) = \sum_{j=0}^{j=2} A_{2j}\psi^j;$$

where here and subsequently $\varphi = \log(H/10^7)$, $\psi = \log(S/10^4)$; if $F_1 < 0$, then $H(P, S) = 10^7$, while if $F_1 \geqslant 0$, then $F_1 = (y_2 + \sqrt{F_1})/2y_3$ and

$$H(P,\ S) = 10^7 e^{F_1};$$

$$F_2 = z_2^2 - 4z_1z_3,$$

$$z_1(H,\ P) = \sum_{i=0}^{i=2} A_{i0}\varphi^i - \lg(P/10^6),$$

$$z_2(H) = \sum_{i=0}^{i=2} A_{i1}\varphi^i \text{ and } z_3(H) = \sum_{i=0}^{i=2} A_{i2}\varphi^i;$$

and if $F_2 < 0$, then $S(H, P) = 10^4$, while if $F_2 \geqslant 0$, then $F_2 = (-z_2 - \sqrt{F_2})/2z_3$ and

$$S(H,\ P) = 10^4 e^{F_2};$$

$$T(H,\ S) = 10^3 \exp\Big[\sum_{i=0,j=0}^{i=1,j=1} A_{ij}\varphi^i\psi^j\Big];$$

$$\sigma(H,\ S) = 10^2 \exp\Big[\sum_{i=0,j=0}^{i=2,j=2} A_{ij}\varphi^i\psi^j\Big];$$

$$\beta(H,\ S) = B \exp\Big[\sum_{i=0,j=0}^{i=1,j=1} A_{ij}\varphi^i\psi^j\Big];$$

$$\rho(H,\ S,\ P,\ T) = \frac{10P}{RT} \exp\Big[\sum_{i=0,j=0}^{i=1,j=1} A_{ij}\varphi^i\psi^j\Big]. \tag{3}$$

We examine the effects of narrowing on MHD characteristics for a diagonal load connection by reference to a transport system. The flow in the channel is described by a system of quasi-one-dimensional magnetohydrodynamic equations [4], viz., the equations of continuity, motion, energy, and state:

TABLE 3. Calculated Parameters for Profiled Section

Parameter	Section number							
	1	2	3	4	5	6	7	8
L, m	0	0,64	1,29	1,93	2,57	3,21	3,86	4,5
P, MPa	0,465	0,0705	0,0410	0,0319	0,0285	0,0275	0,0282	0,0309
$\langle T \rangle$, °K	3098	2571	2457	2405	2374	2353	2341	2344
$\langle U \rangle$, m/sec	2105	2415	2314	2128	1908	1681	1461	1229
$\langle \sigma \rangle$, S/m	66,7	29,2	23,1	20,4	18,6	17,1	16,0	15,5
$\langle \beta \rangle$	0,20	1,27	2,18	2,79	3,14	3,26	3,2	2,93
E_x, V/m	—1015	—2015	—2875	—3253	—3229	—2888	—2415	—1963
γ, deg	28,0	34,5	39,9	45,5	51,8	57,7	63,7	69,6
Q, mW	0	1,935	1,45	1,87	2,23	2,52	2,77	2,93
$\langle F \rangle$, m²	0,0017	0,0088	0,0158	0,0229	0,0299	0,0369	0,0440	0,0510
q, mW/m²	12,2	2,32	1,18	0,724	0,480	0,332	0,238	0,178
y_0, m	0,03	0,15	0,27	0,39	0,51	0,63	0,75	0,87
N, mW	0	0,352	1,03	1,88	2,76	3,56	4,22	4,75
N_V, mW/m³	30	88	93,6	94	84,9	71,9	58,6	50,8

$$\rho U \langle F \rangle = g,$$
$$\rho U \frac{dU}{dx} + \frac{dP}{dx} = \langle j_y \rangle B + F_f ,$$
$$\rho U \frac{d}{dx}\left(H + \frac{U^2}{2}\right) = \langle j_x \rangle E_x + \langle j_y E_y \rangle + Q, \tag{4}$$
$$P = \rho RT.$$

Estimates show that a turbulent boundary layer is produced under these conditions. We assume that the velocity and enthalpy changes in the layer are described by the one-seventh law. For engineering purposes, the friction and heat transfer are satisfactorily described by simple models (models for flow around a flat plate and flow in a tube)

$$F_f = \frac{C_f}{2\rho TD}\rho U^2,\ Q = (q_\text{к} + q_R)\frac{\Pi}{g},$$

where $C_f = 0{,}118\,\mathrm{Re}_x^{-0,2}$; $q_\text{к} = \frac{g}{F}\,\mathrm{St}\,(H^* - H_w)$; $\mathrm{St} = 0{,}029\,\mathrm{Re}_x^{-0,2}\,\mathrm{Pr}^{-2/3}$; $H^* = H + \sqrt[3]{\mathrm{Pr}}\frac{U^2}{2}$.

Radiative heat transfer plays no decisive part for a supersonic flow and can be derived in the bulk-omission approximation by the use of an effective degree of blackness ε:

$$q_R = \varepsilon \cdot 5{,}67 \cdot 10^8 (T^4 - T_w^4).$$

See [5] for the equation systems for a diagonal generator of rectangular cross section.

The formulas for a generator with a nonrectangular cross section are [6]

$$\langle j_y \rangle = \frac{I_y}{\Delta x \langle z_* \rangle} = \frac{\frac{I_\text{н}}{\Delta x \langle z_* \rangle}(\langle B \rangle - \mathrm{ctg}\,\alpha) - \left(\langle U \rangle B - \frac{\Delta V}{y_0}\right)\frac{\langle \sigma z_* \rangle}{\langle z_* \rangle}}{G_\text{В} - \mathrm{ctg}^2\alpha}, \tag{5}$$

$$\langle j_x \rangle = \frac{I_x}{\langle F \rangle} = \frac{\frac{I_\text{н}}{\langle F \rangle}(G_\text{В} + \langle \beta \rangle\,\mathrm{ctg}\,\alpha) - \left(\langle U \rangle B - \frac{\Delta V}{y_0}\right)\frac{\langle \sigma z_* \rangle}{\langle z_* \rangle}}{G_\text{В} + \mathrm{ctg}^2\alpha}, \tag{6}$$

$$E_x = \frac{\frac{I_\text{н}}{\langle F \rangle}(G_\text{В} + \langle \beta \rangle^2) - \left(\langle U \rangle B - \frac{\Delta V}{y_0}\right)\frac{\langle \sigma z_* \rangle}{z_*}(\langle \beta \rangle + \mathrm{ctg}\alpha)}{\langle \sigma z_x \rangle / z_* (G_\text{В} + \mathrm{ctg}^2\alpha)}. \tag{7}$$

The integral characteristics for rectangular and profile sections may be determined by solving the optimization problem for these two types of section

The optimization problem is formulated as follows: we have to find a set of parameters (geometrical dimensions, magnetic-field induction, combustion-chember pressure, etc.) such

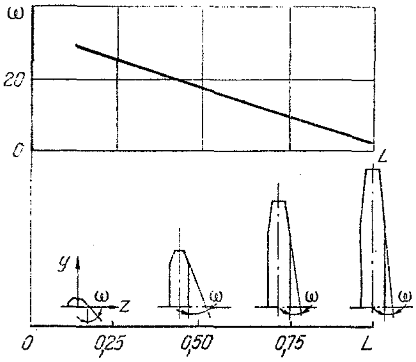


Fig. 1. Change in narrowing angle along the length.

that the target function (generated power or power per unit volume) attains its maximum subject to certain constraints. In the present case, the best demonstration of a profiling effect is obtained by incorporating only one constraint: the flow over the entire channel should not show blocking, i.e., the Mach number should not approach 1 anywhere. The target function Φ is specified inexplicitly. To determine the value with a set of adjustable variables (control vector v), it is necessary to integrate (4), and then determine the parameters over the length of the channel and the integral characteristics. Here we assume that the set of values for the target function with vector argument v is a multidimensional surface that is smooth apart from the numerical determination at Φ. The constraints are incorporated by means of penalty functions [7].

The maximum in the target function is found by gradient-descent and variable-direction methods with automatic switching from one to the other when either is exhausted. Interactive operation is envisaged in the optimization program [8]. The operator can intervene in the optimization by specifying initial values for the control vector and altering the search method. This enables one, on the one hand, to use engineering experience and intuition to find the optimum more rapidly while, on the other, it enables one to demonstrate that the maximum found is global for the relevant case.

The optimization was solved for rectangular and profiled sections. We used the same parameters in both cases: combustion-product flow rate 2.4 kg/sec, stagnation enthalpy at the inlet 8.35 MJ/kg, stagnation pressure at the exit 0.12 MPa, and magnetic induction 2.0 T. The wall temperature was dependent on the fuel type. In our case [3], it is influenced by the molten aluminum oxide depositing on the walls. The wall temperature was taken as 2300°K.

The boundary-layer effect was incorporated only near the electrode walls. The thickness was determined on the basis of the free switching length of 2 m.

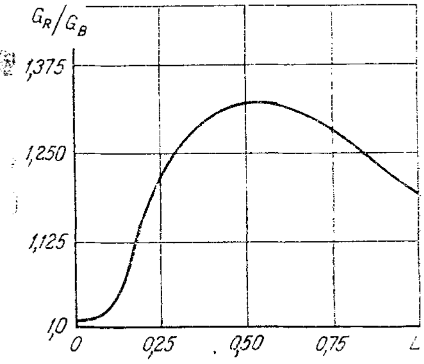
The profiling was performed as in [1] from

$$\frac{\beta j_y}{\sigma} = \text{const.} \tag{8}$$

If (8), i.e., the Hall emf, is kept constant over the entire section, the profile narrows towards the electrode wall following a law almost rectilinear almost throughout the boundary-layer zone, and only near the wall for a distance of less than a millimeter is there section expansion. Such a profile is undesirable and is not feasible, so subsequently we assumed a constant narrowing angle, the more so because the contribution from the zone expanding near the electrode wall to the inhomogeneity parameter is only about 1%.

Some results are given in Tables 2 and 3 respectively for rectangular and profile sections.

Figures 1-3 show the main results on the geometry and electrodynamic characteristics; Fig. 1 shows the variation in the narrowing angle along the length (the total channel length is taken as unit). In the lower part of the figure, we show half the section by virtue of the symmetry together with the deformation along the length. In Fig. 2, the inhomogeneity parameter for the profiled section exceeded the ideal value of one by about 1% over the entire length. The value of G_B of one can also be obtained analytically if one assumes that the Hall parameter is constant over the cross section and that the field strength is transverse along the y axis [5]. Therefore, the curve shows the variation in G_R along the length, whose maximum value is about 1.35.



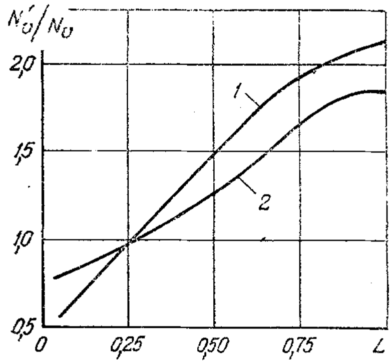


Fig. 2 Fig. 3

Fig. 2. Change in the ratio of the inhomogeneity parameter for the rectangular section to that for the profiled one along the length.

Fig. 3. Distribution along the length of the ratio of the specific power for the rectangular section to that for the profiled one with a variable angle of narrowing (1) and a constant angle (2).

The inhomogeneity parameter increases, which is related to deterioration in the MHD power parameters, and it also influences the curvature of the current lines. This leads to an uneven current-density distribution over the electrodes (the maximum current density occurs at the edges) and reduces the working lives of the electrodes and insulation between them, as well as causing breakdown between electrodes.

Curve 1 in Fig. 3 illustrates the distribution of the ratio of the specific power for the profiled section to that for the rectangular one along the length of the channel.

The integral specific power for the profiled section is more than twice that for the rectangular one, which is particularly important for transport MHD systems, where one of the most important requirements is minimum weight and size. Curve 2 in Fig. 3 shows the same relationship as curve 1 but for the case where the narrowing angle ω is taken as constant at 10° throughout the length. There is here again an integral positive effect; i.e., the narrowing towards the electrode walls in the boundary-layer zones by 10° improves the channel characteristics by about a factor 1.8.

Therefore, the calculations show that it is desirable to profile the section for the combustion products from high-enthalpy fuel. A fixed value for the magnetic induction has been taken for the larger channel volume with the profiled section. In fact, a channel with a smaller working volume can have a higher magnetic induction with the same energy consumption for exciting the field, in accordance with the reduced working volume, which produces an even greater positive effect.

NOTATION

H^*, governing enthalpy; H, static enthalpy; H_w, wall enthalpy; P, pressure; T, flow temperature; T_w, channel wall temperature; U, velocity; ρ, density; R, gas constant; F, cross-sectional area; g, flow rate; Q, heat losses; F_{TP}, frictional force; C_f, coefficient of friction; D, hydraulic diameter; Π, channel section perimeter; q_K, convective heat flux; q_R, radiant heat flux; Re_x, Reynolds number; St, Stanton number; j_x, j_y, projections of current density on x and y axes, respectively; I_x, I_y, currents along x and y, respectively; I_H, load current; E_x, projection of electric field on x axis; σ, conductance; β, Hall parameter; G_R, G_B, nonuniformity parameters; ΔV, electrode voltage drop; Δx, integration step along the x axis; z_*, distance between insulating walls; $y_\circ$, distance between electrode walls; γ, inclination of line connecting electrodes having the same potential; ω, channel taper angle towards electrode wall; N, power; N'_V, N_V, specific power of a profiled cross section and rectangular section, respectively; S, entropy.

LITERATURE CITED

1. A. A. Beloglazov, V. A. Bashkatov, and É. É. Shpil'rain, "Reducing the harmful effects of Hall currents on the characteristics of an MHD generator," Magn. Gidrodin. No. 1, 99-104 (1980).
2. R. J. Rosa, Magnetohydrodynamic Energy Conversion, McGraw-Hill (1968).
3. A. V. Gubarev and Yu. G. Tynnikov, "Flow modes in an MHD generator with strong flow retardation," Proceedings of the Eighth International Conference on MHD Energy Conversion, Moscow, 12-18 September 1983 [in Russian], Vol. 5, IVTAN, Moscow (1983), pp. 89-97.
4. A. B. Vatazhin, G. A. Lyubimov, and S. A. Regirer, Magnetohydrodynamic Flows in Channels [in Russian], Nauka, Moscow (1970).
5. V. M. Batenin and A. A. Beloglazov, "Analysis of a frame MHD generator under particular conditions," in: MHD Generators and Thermoelectric Engineering [in Russian], Naukova Dumka, Kiev (1983), pp. 40-48.
6. Yu. M. Volkov, R. V. Dogadaev, et al., The Effects of Electrode Processes on the Characteristics of Pulsed MHD Generators [in Russian], IAE Preprint 2542 (1975).
7. É. Polak, Numerical Optimization Methods: A Unified Approach [Russian translation], Mir, Moscow (1974).
8. A. L. Shevchenko, "Interactive optimization for power equipment involving discharge in fluids," in: Interactive Solution of Scientific Problems [in Russian], Issue 5, IVTAN, Moscow (1983), pp. 30-41.

NUMERICAL INVESTIGATION OF THE TEMPERATURE FIELD OF A DAM WITH FREEZING COLUMNS

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The problem of interconnected heat and mass transfer is numerically solved. The temperature field of a dam with an artificial freezing antifiltration curtain is determined.

In the construction of dams in the permafrost region the problem of the thermal stability of the soils arises because when they melt, they lose their load-bearing capacity and become highly water-permeable. To maintain the soils in the foundation and in the body of the dam in their frozen state, and thereby also to prevent losses on filtering, soils artificially frozen with the aid of number of boreholes are at present widely used: in some cases by stimulating circulation of natural cold, in others with the aid of refrigeration techniques.

Several authors in the USSR and other countries studied in recent times the processes of freezing of the soil, with filtering of the moisture taken into account. For instance, Melamed and Medvedev [1] studied the process of freezing of finely dispersed soil taking bulging into account on condition that the phase boundary between water and ice the moisture is constant and equal to its lower limit. Frivik and Komini [2] presented the results of experimental and numerical investigation of the process of freezing of the soil with a view to the filtration flows of moisture (the equation of heat conduction with a convective term is solved, where the speeds are determined from Darcy's law) with constant moisture equal to saturation moisture; on the surface of the refrigeration plant, boundary conditions of the first kind are specified.

In the present article we examine the problem of determining the temperature field of a dam with an artificially provided antifiltration curtain. Mass transfer in the melting zone considerably complicates the investigation because it makes it necessary to take into account the interaction of the temperature and moisture fields in the melting and frozen zones of

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